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13. ABSTRACT (Maximum 200 words)

Two separate lines of research have been developed under this contract: (a) numerically induced chaos; and (b) waves in shallow water. In (a), careful numerical studies show how numerical simulations of nonlinear dynamical systems can exhibit chaos, even for a dynamical system that is known not to be chaotic. In (b), explicit analytical models are used to describe waves in shallow water, without the usual restrictions to infinitesimal wave amplitudes or to one-dimensional waves.

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Nonlinear Ocean Waves

Progress Report: 1/1/92-12/31/92

Several lines of independent research were initiated in the first year of this contract. As discussed below, the research breaks naturally into two major topics: (a) numerically induced chaos; and (b) waves in shallow water. No papers have been submitted for publication yet, but that situation should change within the next few months.

(a) Numerically induced chaos

A recurrent theme in recent research has been the discovery that nonlinear dynamical systems can, and often do, behave chaotically. Such discoveries have come in many branches of physics, including the study of ocean waves. In almost all cases of physical interest, the discovery is based on numerical calculations of a discretized version of the dynamical system of interest. The assumption underlying all of this research is that chaos observed in the discrete system implies chaos in the continuous system. The assumption is so basic that often it is not even stated.

Nonetheless, current work of M.J. Ablowitz, B. Herbst, and C. Schober casts doubt on this basic assumption. They have studied a variety of numerical discretizations of the nonlinear Schrödinger equation,

$$i\,\partial_t\psi + \partial_x{}^2\psi + 2|\psi|^2\psi = 0, \qquad \text{(NLS)}$$

with periodic boundary conditions. This equation is a well-known model of waves in deep water. It is known to be completely integrable, and it is known to be nonchaotic. Even so, for the right initial conditions, every discretization of the equation that they have tested has shown temporally chaotic behaviour. Even worse, (10^{-16}) roundoff error has initiated strong instabilities that render numerical calculations meaningless within a few time units. Their work, which soon will be submitted for publication, has potentially important implications for all research on chaos that depends on numerical computation, including numerical simulations of ocean waves.

(b) Waves in shallow water

A major objective of the research under this contract has been to develop an analytical model waves in shallow water, without the usual restrictions to waves of infinitesimal amplitude, or to waves with one-dimensional surface patterns. The theoretical basis for this

work is the fact that the equation due to Kadomtsev & Petviashvili (1970),

$$\partial_x(\partial_t u + u\partial_x u + \partial_x^3 u) + \partial_y^2 u = 0, \qquad \text{(KP)}$$

describes approximately the evolution of waves of moderate amplitude as they propagate primarily in one direction in shallow water. This basis is strengthened by experimental comparisons by Hammack, Scheffner and Segur (HSS; 1989, 1991), who found that the equation remains reasonably accurate even if the waves are not small, and even if their surface patterns are fully two-dimensional.

Because the KP equation is completely integrable, it admits an infinite family of exact solutions, which in turn provides a sequence of increasingly complicated models of wave dynamics in shallow water. The simplest models ("genus 1") are ordinary cnoidal waves. The next level of complication ("genus 2") provides a 6-parameter family of periodic waves with two-dimensional surface patterns. The earlier experiments of HSS were based on this family of models. There are three lines of current research with solutions of genus 2:

(i) When one-dimensional cnoidal waves (of genus 1) reflect obliquely off a wall, they generate two-dimensional periodic waves of genus 2. This situation is common, so it is important to be able to predict the outgoing waves (of genus 2) from information about the incoming waves (of genus 1). The experiments have already been completed, and J.L. Hammack is carrying out the calculations needed to implement the theory.

(ii) D. McCallister & H. Segur are developing an algorithm (*i.e*, a computer program) to fit a 6-parameter, KP solution of genus 2 to any two-dimensional, periodic wave in shallow water. They are calibrating their algorithm by comparing to laboratory data that Hammack & Scheffner had generated in earlier experiments..

(iii) Once this algorithm is finished, they intend to apply it to ocean data taken at the field station run by the Army Corps of Engineers (CERC) at Duck, N.C. Because these are field data, they probably contain few waves that are strictly periodic, so this will be an important test of the robustness of the model.

At the next level of complication ("genus 3"), the waves are only quasi-periodic, and they are inherently unsteady. For waves this complicated, neither theory nor experiments have been investigated to any extent, by us or by anyone else. Preliminary work is proceeding along three lines:

(i) Based on earlier work by B.A. Dubrovin (1981), Dubrovin and Segur are developing an effective theory to describe KP solutions of genus 3. It is a general problem that much of the existing work on

higher genus solutions of KP is not "effective", in the sense that it is not constructive, and/or it cannot be implemented in a useful algorithm.

(ii) J.L. Hammack is designing and building a multicomponent wavemaker system with distributed computer control of paddle positions and velocities. When completed, the system will be capable of performing the experiments needed to test the KP solutions of genus 3. Initial design for a single component of the system has been completed; some equipment has been purchased and tested, including a Parker-Compumotor servo-drive, brushless DC motor, and power transformer. The next step involves the design, purchase and testing of a computer-based controller for the motor-drive system of a single wavemaker component.

(iii) An essential feature of the KP solutions of genus 2 is their observed stability. Solutions of higher genus may or may not be as stable as those of genus 2. J.H. Curry is formulating a method to test the stability of solutions of higher genus, and to determine the practical consequences of an instability if one appears.

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